Narrow-Band Rejection Filter for Suppression of the Concentrated Harmonic Noise based on Discrete Fourier Transformation

Leonid Fainzilberg, Galina Glushauskene
International Research and Training Center of Information Technologies and Systems
Prospect Academica Glushkova, 40, Kiev-680, Ukraine, 03680
fainzilberg@voliacable.com

Abstract

The modified method of stationary harmonic noise filtration on the basis of discrete Fourier transformation is offered.

This method is especially effective if the noise is concentrated in useful signal frequencies, when using well-known rejection filters can deform the useful signal in consequence of spectrum leakage effect. The mathematical justification of the offered improvements of this method, maximally saving the form of useful signal is given. The results of experiments confirmed that such procedure allows to effectively suppress of the concentrated frequency noise without substantial distortions of informative fragments of useful signal even in those cases, when frequency of noise is concentrated in the interval of the loaded frequencies of useful signal.

1. Introduction

In signal processing one often has to deal with additive harmonic noise, superimposed on the signal [1, 2]. In cases when frequencies of useful signal and noise are essentially different, we use different rejection filters[3,4], stop band of which is adjusted for harmonic noise cancellation.

However, if the noise is concentrated in useful signal frequencies, using well-known rejection filters can deform the useful signal. This is inadmissible in deciding some practical tasks, in particular in computing physiological signals, such as electrocardiogram, rheogram, magnetocardiogram etc.

The feature of such signals is that valuable information about object state is concentrated in small fragments of signal range of definition. Therefore, if the filtration causes even insignificant distortions of informing fragments shape, such distortions can result in erroneous diagnostics [5,6].

The modified method of stationary harmonic noise filtration on the basis of discrete Fourier transformation is offered, and also the mathematical justification of the offered improvements of this method, which maximally saves the form of useful signal .

2. Problem statement

Let the observable signal be sequence of discrete values $y[k] \equiv y(t_k)$ in equidistant time points $t_k \equiv k\Delta$, $k = 0,1,..,K-1$, where Δ is a time quantization step. It is assumed that the observable signal $y[k] = y_0[k] + h[k]$ is additive mixture of useful signal $y_0[k]$ and noise $h[k]$ as a sum of stationary harmonic oscillations

$$h[k] = \sum_{g=1}^{G} a_g \cos(2\pi f_g k\Delta + \phi_g), \qquad (1)$$

here a_g, f_g, φ_g are amplitude, frequency and initial phase of g-th harmonic component. Frequencies of harmonic components are known accurate within contingencies $f_g^{\min} \le f_g \le f_g^{\max}$,and at that turn-downs can coincide with the areas of the loaded frequencies of useful signal.

A problem is set to design a rejection filter, providing effective harmonic noise (1) cancellation with minimum distortions of useful signal $y_0[\cdot]$ form.

3. Block filtration of harmonic noise

Let's consider one of known approaches to harmonic noise suppression [3], based on direct and reverse discrete Fourier transformation (DFT).

It is known that direct DFT allows to decompose a signal $y[k]$, fixed in K points of the bounded time interval $K\Delta$ (in seconds) with the quantum Δ, into K harmonic components. At that by virtue of linearity of Fourier transformation for an additive noise the spectrum $S\{y\} = S\{y_0\} + S\{h\}$ of the observed signal $y[\cdot]$, built on the basis of direct DFT, is equal to the sum of spectrums $S\{y_0\}$, $S\{h\}$ of useful signal $y_0[\cdot]$ and noise $h[\cdot]$. Hence, it would seem, for the stationary harmonic noise (1) rejection, it is enough to suppress spectrum components $S\{h\}$, corresponding to noise $h(\cdot)$ proper frequencies , and using mutual reversibility of direct and reverse DFT procedures to conduct reverse transformation on the basis of reverse DFT for renewal of useful signal $y_0[\cdot]$ in time region.

However, such attractive trick is not always effective. The point is that in general case the spectrum of single harmonic oscillation, built on the basis of direct DPF, «distributes» on a number of contiguous frequencies, i.e. we observe spectrum leakage. Therefore, if while filtrating noise to reject all DFT components, on which the noise burden will be distributed during spectrum leakage, at near frequencies of noise and signal it will inevitably be necessary to affect the components bearings information about the useful signal. As a result, after reverse Fourier transformation the useful signal will be distorted.

As the effect of spectrum leakage results from the nature of DFT, let's concentrate on the problem. For this purpose let's consider a simple discrete harmonic signal, so let's write (1) for $G=1$ in such way:

$$h[k] = a\cos(\omega k\Delta + \varphi), \quad k = 0,...,K-1, \quad (2)$$

here a is amplitude, Δ – time quantum, ω – circular frequency, φ – initial phase.

Using direct DFT procedure, let's represent the signal $h[k]$ in such mode:

$$Y_n = \sum_{k=0}^{K-1} y[k]e^{-2\pi ink/K} = \sum_{k=0}^{K-1} a\cdot\cos(\omega k\Delta+\varphi)\cdot e^{-2\pi ink/K}.$$

or, taking account of Euler formulas:

$$Y_n = \frac{a}{2}\cdot e^{\phi i}\cdot\sum_{k=0}^{K-1} e^{ki\left(\omega\Delta-\frac{2\pi n}{K}\right)} + \frac{a}{2}\cdot e^{-\phi i}\cdot\sum_{k=0}^{K-1} e^{ki\left(-\omega\Delta-\frac{2\pi n}{K}\right)}, \quad (3)$$

or in that way:

$$Y_n = \frac{a}{2}\cdot e^{\phi i}\cdot\frac{1-e^{i(K\omega\Delta-2\pi n)}}{1-e^{i(\omega\Delta-2\pi n/K)}} + \frac{a}{2}\cdot e^{-\varphi i}\cdot\frac{1-e^{i(-K\omega\Delta-2\pi n)}}{1-e^{i(-\omega\Delta-2\pi n/K)}}. \quad (4)$$

As we can see, Y_n can be equal zero if and only if $(\pm K\omega\Delta - 2\pi n) = 2\pi M$, or $K\omega\Delta/2\pi = M$, here M is integer. In this condition $Y_n = 0$ for such values n when $e^{i(\pm\omega\Delta-2\pi n/K)} \neq 1$. In the case when $e^{i(\pm\omega\Delta-2\pi n/K)} = 1$, it's easy to prove that:

$$\frac{1-e^{i(\pm K\omega\Delta-2\pi n)}}{1-e^{i(\pm\omega\Delta-2\pi n/K)}} = K.$$

Thus, if $K\omega\Delta/2\pi$ is an integer, $Y_n = \frac{aK}{2}e^{\pm i\varphi}$ for such values n which satisfy the condition $e^{i(\pm\omega\Delta-2\pi n/K)} = 1$, and for another values n $Y_n = 0$.

It is obvious, that $e^{i(\pm\omega\Delta-2\pi n/K)} = 1$ if and only if the value $\pm\omega\Delta - 2\pi n/K$ is multiple to 2π. Taking into account the limitations $0 < n \le K$, we can see that in case when $K\omega\Delta/2\pi$ is an integer, non-zero amplitudes Y_n emerge in expression (4) only when

$$n = K\frac{\omega\cdot\Delta}{2\pi} \quad \text{or} \quad n = K\left(1-\frac{\omega\cdot\Delta}{2\pi}\right).$$

As a result, from formula (4) we get that in case when the relation $K\omega\Delta/2\pi$ is an integer, the calculated DFT contains just two non-zero spectral values:

$$Y_n = \begin{cases} \frac{aK}{2}e^{i\varphi}, \text{ when } n = \frac{\omega\Delta}{2\pi}K, \\ \frac{aK}{2}e^{-i\varphi}, \text{ when } n = (1-\frac{\omega\Delta}{2\pi})K, \\ 0, \quad \text{otherwise.} \end{cases} \quad (5)$$

But if the relation $K\omega\Delta/2\pi$ is not an integer, $Y_n \neq 0$ for any value n. This is the effect of spectrum leakage of single harmonic oscillation (2), represented by the finite number of discrete values.

It's clear that for constructing an effective harmonic noise (1) suppression procedure it is necessary to eliminate or at least weaken the effect of spectrum leakage of discrete harmonic signal. A known technique of weakening the spectrum leakage effect is multiplying the signal by a weight function which has its maximum in the central point of signal (for $k = K/2$) and fluently decreases to its edges ($k=0$ and $k = K-1$). In this case the formula of DFT takes the form:

$$Y_n(W) = \sum_{k=0}^{K-1} W[k]y[k]e^{-i\frac{2\pi nk}{K}}.$$

It is considered that in case of a good choice of $W(k)$ it is possible to weaken in a way spectrum leakage effect. However, the inevitable paying for the use of weighting function $W(k)$ is distortion of useful signal spectrum.

Because of it there is a natural desire to find the method of resisting the effect which generates the spectrum leakage effect, but not only its consequences. In other words, we'll make an effort to get the «adequate» spectrum of discrete harmonic oscillation on the basis of traditional DFT algorithm without using of weighting function $W(k)$.

4. Background of the offered algorithm

Let's use (4) and represent the amplitudes of spectrum density as a function of a real variable k :

$$Y_n(k) = \frac{a}{2}\cdot e^{\phi i}\cdot\frac{1-e^{i(k\omega\Delta-2\pi n)}}{1-e^{i(\omega\Delta-2\pi n/k)}} + \frac{a}{2}\cdot e^{-\varphi i}\cdot\frac{1-e^{i(-k\omega\Delta-2\pi n)}}{1-e^{i(-\omega\Delta-2\pi n/k)}}.$$

Let's suppose that $K\cdot(\omega\Delta/2\pi) = R$, where R isn't integer. In this case, as we have shown before, spectrum leakage effect is obligatory observed. But for $R > 1$ there is such a value $k < K$ (not obviously integer), for which $k \cdot (\omega\Delta / 2\pi) = \lfloor R \rfloor$, where $\lfloor \eta \rfloor$ means the integer part of η.

Thus, if to define K_0 as:

$$K_0 = \sup\{k : \ k < K, \ k \cdot (\omega\Delta / 2\pi) - \textbf{integer}\} = \\ = \lfloor R \rfloor \cdot 2\pi / \omega\Delta \quad (6)$$

then by virtue of continuity of function $Y_n(k)$ on the interval $[K_0, K]$:

$$\lim_{k \to K_0} Y_n(k) = Y_n(K_0).$$

As $K_0 \cdot (\omega\Delta/2\pi) = \lfloor R \rfloor$ is an integer, for $k = K_0$ the condition of spectrum leakage effect absence that we have received earlier is fulfilled. Hence, if K_0 is an integer, with diminishing the interval of observation length from K to K_0, gradual weakening of spectrum leakage effect and its complete disappearance for the value of length K_0 occurs, i.e.

$$\lim_{k \to K_0} Y_n(k) = Y_n(K_0) = \begin{cases} \frac{aK_0}{2} e^{i\varphi}, & when \ n = \frac{\omega\Delta}{2\pi} K_0, \\ \frac{aK_0}{2} e^{-i\varphi}, & when \ n = (1 - \frac{\omega\Delta}{2\pi}) K_0 \\ 0, & \textbf{otherwise.} \end{cases}$$

However, the case when K_0, determined by (6), is an integer is rather a rare exception then a rule. It's clear, that if K_0 is not integer, fully removing the spectrum leakage effect is not succeeded. At the same time, if the interval observation length is equal to the nearest integer value to K_0, the spectrum leakage effect is less than for the primary length K.

One can notice that the earlier formulated condition of spectrum leakage effect absence is equivalent to the condition $\omega\Delta / 2\pi = M / K$, where M is integer. Thus, it is possible to formulate next statement.

Statement 1. If unknown circular frequency ω of a discrete harmonic signal and the time quantization step Δ satisfy the condition $\omega\Delta / 2\pi = p / q$, ($p/q$ - rational number), then for the signal length K multiple to denominator q of this number, the spectral decomposition of the signal on the DFT basis will have only two non-zero components.

Really, if $\omega\Delta / 2\pi = p / q$, then for K multiple to q, i.e. $K = m \cdot q$, we have $K\omega\Delta / 2\pi = m \cdot p$ - an integer and, as we mentioned earlier, the spectrum leakage effect is absent.

Corollary. If the unknown circular frequency ω of a discrete harmonic signal and the time quantization step Δ are such that $\omega\Delta / 2\pi = p / q$, and p / q is some rational number, the denominator q of which is less then the length K of the processed signal, then for diminishing the number of processed points we'll have a periodic (with a period q) weakening of spectrum leakage effect up to its complete removal.

Really, if $K = nq + m \ (m < q)$, then the expression

$$K_1 \omega\Delta / 2\pi = nq \cdot p / q = np$$

is an integer for the value $K_1 = K - m = nq$, and therefore for such signal length the spectrum leakage effect is absent. At the further diminishing of signal length for such values:

$$K_2 = K - m - q, \ldots, \ K_i = K - m - (i-1)q = (n+1-i)q$$

the values

$$K_i \omega\Delta / 2\pi = (n+1-i) \cdot q \cdot p / q = (n+1-i)p$$

are integer and, therefore, the periodically removal of spectrum leakage effect with a period q can be observed.

It's obvious that the case when $\omega\Delta / 2\pi$ is a rational number is a rare exception. In general case $\alpha = \omega\Delta / 2\pi$ is rather an irrational number. In this case we have a natural desire to make attempt to approximate $\omega\Delta / 2\pi$ by some rational number.

In number theory there is a set of effective methods for approximating real numbers by rational fractions, in particular approximation by continued fractions and approximants. According to the Dirichlet theorem [7], for any real number α and arbitrary number $\tau > 1$ it is possible to find a rational fraction $\frac{a}{b}$ such, that:

$$\left| \alpha - \frac{a}{b} \right| < \frac{1}{b\tau}, \ b \le \tau.$$

As the fraction, satisfying the condition of the Dirichlet theorem, it is accepted to use the so-called n-th approximant $\frac{P_n}{Q_n}$ of decomposition α to a continued fraction. Here n is maximal number, for which $Q_n < \tau$. For example, for approximating an irrational number $\sqrt{5}$ by rational fraction $\frac{a}{b}$ accurate within $\frac{1}{1000\,b}$ one has to decompose $\sqrt{5}$ into continued fraction $\sqrt{5} = [2,4,4,4,4,4...]$ and evaluate approximant for this decomposition: 2/1, 9/4, 38/17, 161/72, 682/305 (further denominators are more than 1000). Then the desired fraction is $\frac{682}{305}$, at that $\left| \sqrt{5} - \frac{682}{305} \right| < \frac{1}{1000 \cdot 305}$.

Thus, according to the Dirichlet theorem, $\alpha = \omega \cdot \Delta / (2\pi)$ can be approximated by some rational number $\frac{p}{q}$ so, that $\left|\alpha - \frac{p}{q}\right| < \frac{1}{q\tau}$ for any $\tau > \mathbf{1}$.

So, if we take $K = M \cdot q$, ($M << \tau$), then

$$\left|K \cdot \alpha - M \cdot p\right| = \left|M \cdot q \cdot \alpha - M \cdot q \cdot \frac{p}{q}\right| =$$

$$= M \cdot q \cdot \left|\alpha - \frac{p}{q}\right| < M \cdot q \cdot \frac{1}{q \cdot \tau} = \frac{M}{\tau},$$

where $M << \tau$.

Hence, if $\omega \cdot \Delta / (2\pi) = \sqrt{5}$, then for interval length K=1000 and given accuracy 1/1000 with diminishing the length K for the values K_1= 915, K_2= 610, K_3= 305 we obtain $K_i \cdot \omega \cdot \Delta / (2\pi)$ as almost integer with accuracy $0.001\, i$, $(i = 1,2,3)$. As a consequence of this fact noticeable reducing of spectrum leakage effect is observed.

According to the experiments, with diminishing the observation interval length the removal of spectrum leakage effect occurs with a much less period. The point is that the approximant is not the unique possibility of approximating the real number by rational fractions. According to the Hurwitz theorem [7], for any real number exist an endless great amount of rational fractions such that:

$$\left|\alpha - \frac{a}{b}\right| < \frac{1}{\sqrt{5} \cdot b^2}.$$

Thus, it is always possible to approximate irrational (in general case) number $\alpha = \omega \cdot \Delta / (2\pi)$ by some rational number with the sufficient accuracy grade, and so to decrease the spectrum leakage effect.

For known values ω and Δ it is sometimes possible to define analytically an optimal value K_{opt} which implies the absence of spectrum leakage.

Statement 2. If circular frequency ω of harmonic noise and time quantization step Δ are known, and $\alpha = \omega \cdot \Delta / (2\pi)$ is a rational number, i.e. $\alpha = p / q$ ($q < K$), then the optimal value K_{opt}, which implies the absence of spectrum leakage, is determined as

$$K_{opt} = \lfloor K / q \rfloor \cdot q.$$

Really, let $K = aq + b$ ($b < q$). Then $K_{opt} = \lfloor K / q \rfloor \cdot q = aq$, and the number

$$K_{opt} \omega \Delta_t / 2\pi = K_{opt} \cdot p / q = aq \cdot p / q = ap$$

is an integer, i.e. according to Statement 1 the spectrum leakage effect is absent.

It's clear that for unknown frequency ω it is impossible to define K_{opt} analytically, moreover the algorithm of calculating the best approximation by rational fractions is too unhandiness. And so all aforesaid serves only as the theoretical ground for the searching procedure offered below.

5. Offered algorithm of harmonic noise suppression

As it is assumed that the noise $h_g[\cdot]$ is concentrated on some fixed frequency $f_g \in [f_g^{\min}, f_g^{\max}]$, but as a result of spectrum leakage its power is allocated on a number of contiguous frequencies, then with weakening of this effect the amplitude of DFT spectral component, which frequency is near to f_g, inevitably increases .

So, if to design a spectrums assemblage $S_K\{\}, S_{K-1}\{\}, S_{K-2}\{\},...$ then for gradually decreasing length K of the processed array there is such a spectrum among this assemblage for which the spectral component, corresponding to frequency f_g, is maximal. This spectrum will define the optimal value K_{opt}.

On fig. 1 the examples of spectrums built on the basis of DFT for harmonic oscillation $h[k] = \cos(2\pi f_g k \Delta)$, $k = 0,..., K-1$ with frequency $f_g = 20.127$ Hz ($\Delta = 0.001$ sec.) for different values of the processed array length K are shown. It's easy to see that with gradual decreasing the number of processed points from $K = 3000$ to $K = 2970$ the spectrum leakage effect weakens in the beginning and then increases again. The spectral amplitude is maximal for optimal value $K_{opt} = 2981$.

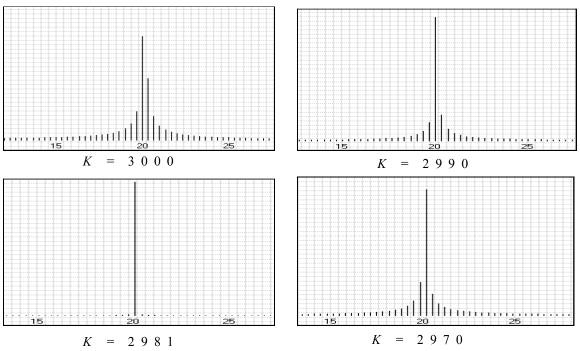

Figure 1. Spectrums of signal $h[k] = \cos(2\pi f_g k \Delta)$, $f_g = 20.127$ Hz, $\Delta = 0.001$ sec for different values of the length K

As in practice the circular frequency ω is unknown as a rule, for achieving the formulated aim using a searching procedure for optimal automatic selection of the number $K_{opt} \le K$ is suggested. To accelerate the number searching it is necessary to take limitations $f_g^{\min}$, $f_g^{\max}$ into account the limits of frequencies interval on which the harmonic noise is probably concentrated, and also to set some possible decreasing of number of points δ_K in the processed array. As a result we come to the next algorithm.

Step 1. Consistently we abbreviate the length of the array, decreasing the number K_j of points being processed from an initial value K to $K-\delta_K$.

Step 2. Using direct DFT procedure, we design the assemblage of spectrums $S_K\{\}$, $S_{K-1}\{\}$, $S_{K-2}\{\}$ for different values K_j , i.e. we calculate the spectral components $C_n(K_j)$, $n=-K_j/2,...,0,...,K_j/2$ (here $C_n(K_j)$ means $|Y_n|$ for the signal length $K_j \in [K-\delta_K, K]$).

Step 3. By the assemblage of built spectrums we determine the optimum value K_{opt} which satisfies the condition:

$$K_{opt} = \underset{K_j \in [K-\delta_K, K]}{\arg\max} \left\{ \frac{\max\limits_{n\in\Omega_h} C_n(K_j)}{\sum\limits_{n\in\Omega_h} C_n(K_j)} \right\},$$

where Ω_h is a set of numbers of spectrums with frequencies in the interval $[f_g^{\min}, f_g^{\max}]$.

Step 4. As an estimation of the frequency $\hat{f}_g$ of harmonic noise we take the value corresponding to the maximal spectral component on the interval $[f_g^{\min}, f_g^{\max}]$, calculated for K_{opt} .

Step 5. We modify the spectrum by making zero the components corresponding to the found frequency $\hat{f}_g$ and the «negative» frequency $-\hat{f}_g$.

Step 6. Using reverse DFT procedure, we restore the useful signal in the temporary realm according to the modified spectrum.

We will show the efficiency of the offered algorithm on the example of filtration of a real electrocardiogram (fig. 2).

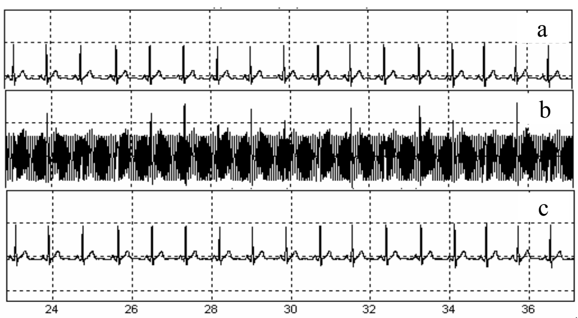

Figure 2. Frequency filtration of ECG useful signal (a), noisy signal (b), result of filtration (c)

It is known that electrocardiogram processing is based on analyzing the form of the informative fragments, reflecting the stages of excitation of separate areas of cardiac muscle (fig. 2, a). However, as a result of imposition of harmonic noise with frequency 16.68 Hz, amplitude of which was 50% turn-down useful signal, the observed signal (fig. 2, b) is substantially distorted and it is impossible to conduct the detailed analysis of the forms of its fragments.

At that the noise frequency 16.68 Hz is located in the 0 – 85 Hz area of the useful signal loaded frequencies (fig. 3, a), and because of spectrum leakage the basic power of the noise is allocated on the interval 12–22 Hz (fig. 3,b).

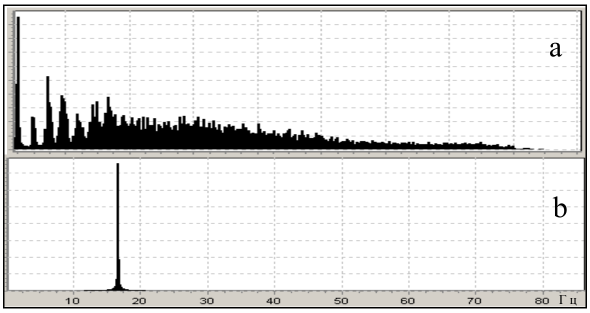

Figure 3. Spectrums of useful signal (a) and harmonic noise 16.48 Hz (b)

As the harmonic noise is additive and the Fourier transformation is linear, it is possible to examine separately the spectrums of the useful signal and the noise. For gradual decreasing of the processed array length from the initial size $K = 30000$ points redistribution of the loaded frequencies of both the noise and the useful signal occurs (fig. 4).

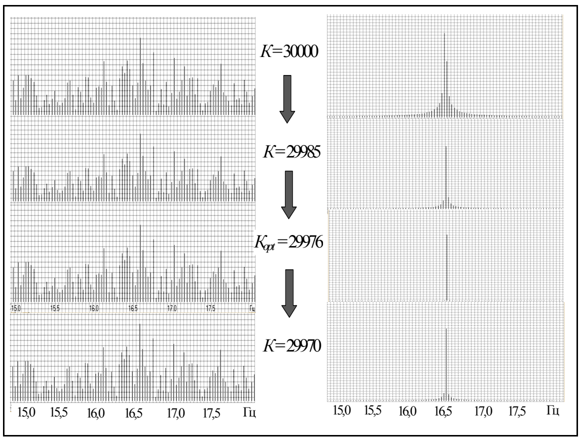

Figure 4. Assemblages of spectrums of the useful signal (on the left) and the additive noise (on the right) for different numbers K of points of the processed signal.

Redistribution of the loaded frequencies of the useful signal spectrum (fig. 4, on the left) does not result in any unpleasant consequences, as by virtue of DFT convertibility it is always possible to restore exactly the useful signal using the abridged array. The main effect of the offered algorithm comes to diminishing the spectrum leakage of the harmonic noise. As soon as the array length diminished to the value of $K_{opt} = 29976$ points, the spectrum leakage effect turned out to be minimal (fig. 4, on the right). It enables us to shorten the rejection band up to suppressing just one of the spectral components of DPF which coincides exactly with the noise frequency. Other harmonics are not affected, that ensures conservation of the useful signal (fig. 2, c).

6. Numerical experiments

The numerical estimation of the efficiency of the offered filtration algorithm was conducted for model signals of different standard forms. During the implementation of the filter the procedure of fast Fourier transformation (FFT) by Mateo Frigo and Steven Johnson [8], which is acknowledged to be one of the best ones, was used.

Let us present the result of filtration of a meander which was generated in 30 thousand of points with digitizing rate F_D=1000 Hz and was distorted by a harmonic noise with frequency 18.1 Hz of different levels – 20 %, 50 % and 100 % of the useful signal size of changing (fig.5).

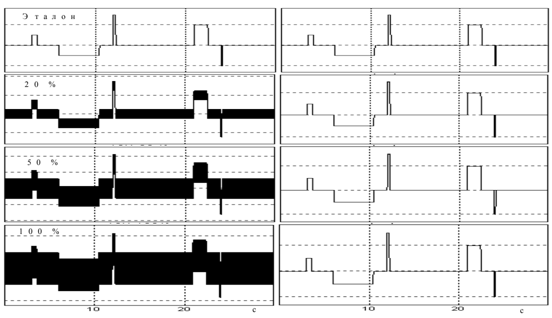

Figure 5. Results of the meander filtration: on the left – before the filtration, on the right – after the filtration

The experiments showed that even for a considerable level of noise it is possible to restore the form of such a difficult signal practically without distortions. So, in particular, after filtrating a noise of 100 % level the divergence between the filtered signal and the etalon one did not exceed 0.22 %.

The efficiency of the offered algorithm was experimentally confirmed during the filtration of other signals.

The developed computational module of a narrow-band rejection filter was used for realization of some application systems, in particular, the **FASEGRAPH**™ system for operative estimation of the functional state of the cardiovascular system of a human by the phase portrait of an ECG [9].

7. Conclusion

The theoretical analysis fulfilled in the article shows that the efficiency of sectional filtration of the concentrated harmonic noise on the basis of direct and reverse DFT can be heightened substantially due to the use of an additional searching procedure which allows to weaken the spectrum leakage effect of harmonic noise. The results of experiments confirmed that such procedure allows to suppress effectively the concentrated frequency noise without substantial distortions of informative fragments of the useful signal even in those cases when the noise frequency is concentrated in the interval of the loaded frequencies of useful signal.

8. References

[1] Oppenheim A.V., Schafer R.W. Discrete-Time Signal Processing, Upper Saddle River, NJ: Prentice-Hall, 1999.– P.– 468-471.

[2] Antoniou A. Digital Filters: Analysis, Design, and Applications. McGraw-Hill, New York, NY, 1993. – 320 p.

[3] Milstein L. Interference rejection techniques in spread spectrum communications, Proceedings of the IEEE, 1988.– Vol. 76. – P. 657-671.

[4] Hsu F., Giordano *A*. Digital whitening techniques for improving spread spectrum communications performance in the presence of narrowband jamming and interference // IEEE Trans. Communications, 1978.– Vol. 26.– No. 2.– P. 209–216.

[5] Malik M, Camm A. J. Components of heart rate variability: what they really mean and what we really measure // American Journal of Cardiology. – 1993. – No. 72. – P. 821-822.

[6] Srikanth T., Napper S.A., Gu H. Bottom-up approach to uniform feature extraction in time and frequency domains for single-lead ECG signal // International Journal of Bioelectromagnetism.– 2002.– Vol. 4. – P. 21-29.

[7] Chandrasekharan K. Introduction to Analytic Number Theory, Springer-Verlag, Berlin-Heidelberg-New York, 1968, 140pp.

[8] Frigo M., Johnson S.G. FFTW: An adaptive software architecture for the FFT, Proc. of the IEEE Intern. Conf. on Acoustics, Speech, and Signal Processing, Seattle, 1998: WA.– Vol. 3.– P. 1381-1384.

[9] Fainzilberg L.S. Nowa metoda interpretacji zapisu EKG w balaniach skriningowych oraz w opiece domowej // Zdrowie publiczne (Public health), 2005.– Vol. 115.– No. 4.– P. 458-464.