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PHASE PORTRAIT OF ELECTROCARDIOGRAM AS A MEANS OF BIOMETRY

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Abstract. *The author develops an approach to constructing biometric systems based on the analysis of the phase portrait of a single channel electrocardiogram (ECG) of the test subject. The rules for solving the problem of human identification and verification (authentication) are proposed. Experimental studies have shown that the proposed decision rules provide 96.6% reliability of identification in the analysis of 3,133 ECGs of 167 individuals and 99.5% reliability of verification in the analysis of 204 ECGs of 62 different individuals. Prospects for further research in solving practical biometric problems are outlined.*

Keywords: *electrocardiogram, phase portrait, biometric system, Hausdorff distance, identification and verification of personality.*

INTRODUCTION

The problem of biometric human identification is based on the analysis of individual characteristics of the person and is becoming increasingly important. Today, the most common are fingerprint systems [1, 2], which recognize the characteristic features of fingerprints. Biometric identification systems are also generally recognized, which are based on the recognition of individual characteristics of the iris of the eye [3], voice [4], and face [5] or its individual parts.

The rapid development of digital computing and smartphone technology has contributed to commercial biometric identification systems that provide a fairly high performance. At the same time, it is known that the unique characteristics on which these biometrics are based can be forged by an attacker. For example, a fingerprint can be forged with a special glove or model (spoofing in fingerprints). Therefore, there is a need for special additional means of protection [6, 7]. Similar forgeries are possible in biometric systems, which are based on the recognition of the iris of the eye, face, and voice.

Thus, scientists are constantly looking for new directions in constructing biometrics systems based on the analysis of such individual human characteristics that are difficult to falsify. One of them is the electrocardiogram (ECG), which provides information about electrical activity of the heart.

The purpose of the article is to investigate the possibilities of ECG to solve problems of human identification and verification.

PROPERTIES OF ECG PHASE PORTRAIT

In [8], Ukrainian scientists, for the first time, formulated a hypothesis about a possibility of identifying a person by ECG. The hypothesis took into account the results of the first experiments, which demonstrated the individual features of the graphic image of the ECG. However, it was not until six years later that a foreign publication [9] appeared on a new method of identifying a person by ECG.

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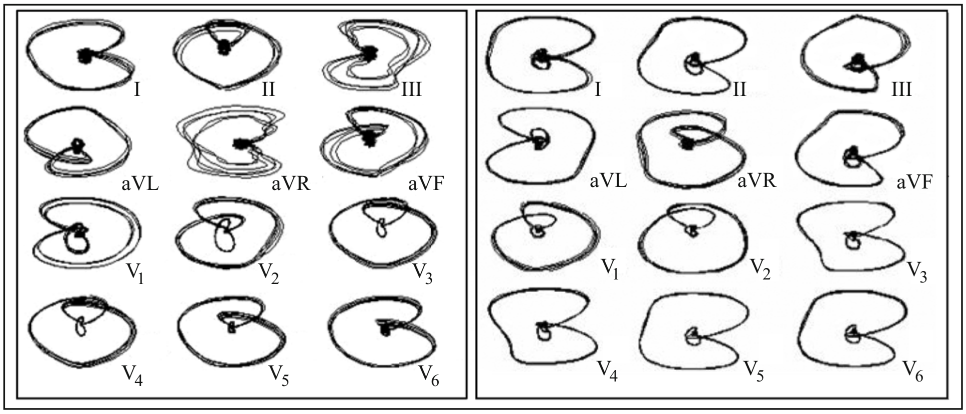

Fig. 1. ECG phase portraits of two persons in 12 traditional leads.

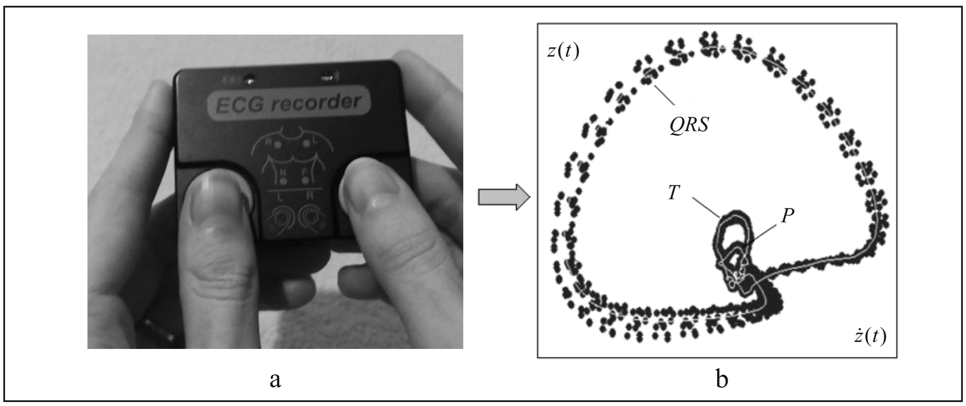

Fig. 2. Formation of a phase portrait of a single channel ECG on the plane $z(t), \dot{z}(t)$.

Subsequently, the method was actively developed in numerous studies by other authors [10–24]. In these works, analysis of ECG signals in the time domain was performed using short-time Fourier transform, wavelet transform, Radon transform, Kalman filter, and other mathematical methods.

Studies have shown (Fig. 1) that the individual features of ECG are more pronouncing when displaying on the phase plane $z(t), \dot{z}(t)$ with the coordinates $z(t)$ (signal of electrical activity of the heart) and $\dot{z}(t)$ (the rate of change of this signal).

Since the simultaneous signal registration in several leads (in the chest leads $V_1, \ldots, V_6$, especially) is inconvenient for solving biometric problems, consider a more practical way to record a single channel ECG in only one lead, namely, the first standard (left and right hands).

Using a microprocessor recorder with finger electrodes (Fig. 2a), the digital sequence $z(t_k)$ of the signal values $z(t)$, which are observed in discrete moments $t_k \equiv k\Delta$, $k = 1, \ldots, K$, of time (Δ is the quantization step), is entered into the computer via Bluetooth and pre-processed, i.e., isoelectric line drift extracting, frequency-selective filtering, and adaptive smoothing are present. Based on the procedure of numerical differentiation with a corresponding regularization procedure, it is possible to obtain an acceptable estimate of the derivative $\dot{z}(t_k)$ in the discrete moments of time t_k. The result is the ECG phase portrait ECG PhP), i.e., a sequence of two-dimensional vectors (points)

$$(z(t_1), \dot{z}(t_1)), (z(t_2), \dot{z}(t_2)), \ldots, (z(t_K), \dot{z}(t_K)),$$

lying on the plane $z(t), \dot{z}(t)$ in the form of a periodic attractor, which displays characteristic loops of the corresponding fragments of the ECG (*P* and *T* waves and *QRS* complex, see Fig. 2b).

We will distinguish two formulations of the biometrics problem.

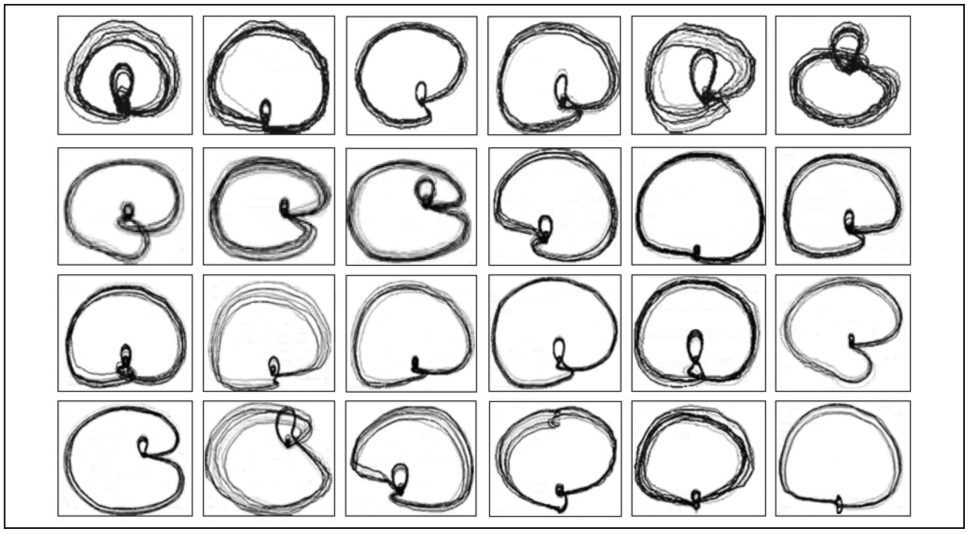

Fig. 3. Examples of phase portraits of single channel ECGs of 24 people.

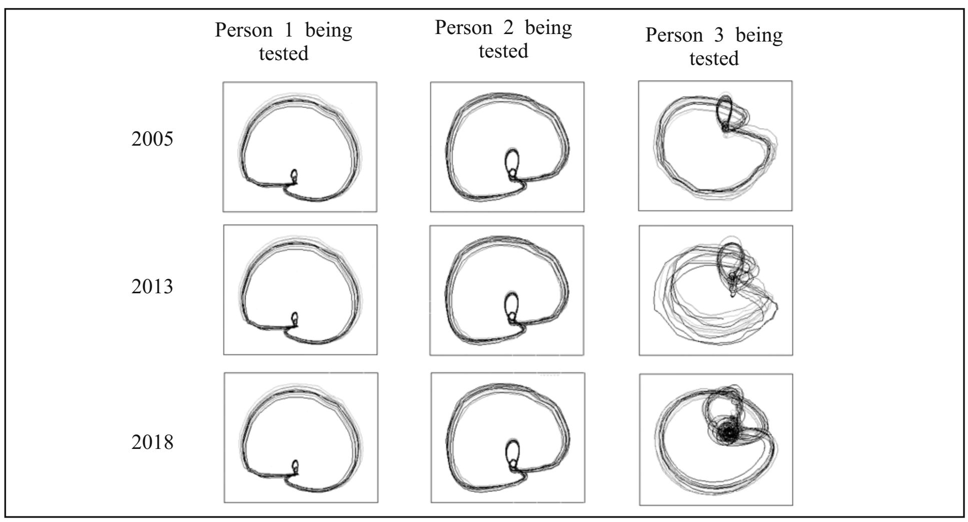

Fig. 4. Dynamics of phase portraits of single channel ECGs of three persons.

Identification Problem. There is a limited list $B = \{B_1, \dots, B_M\}$ of persons being tested. It is necessary to determine the number $m = 1, \dots, M$ in the list $B = \{B_1, \dots, B_M\}$ using the automatic processing of the ECG of one person $B_m \in B$.

Verification (Authentication) Problem. The person being tested reports his (her) login, which will determine his (her) number $m = 1, \dots, M$ in the list $B = \{B_1, \dots, B_M\}$. It is necessary to confirm or deny this login using automatic processing of the ECG PhP ("smart" password). Moreover, it is assumed that the name of this person may not be in the list $B = \{B_1, \dots, B_M\}$.

Long-term observations have shown that phase portraits of single channel ECGs, as well as fingerprints have individual features (Fig. 3) (unchanged over a long period of time (Fig. 4), unless, of course, there was no serious organic heart disease during this period). Given these facts, biometric systems based on the analysis of ECG PhP were constructed.

HUMAN IDENTIFICATION BY PHASE PORTRAIT OF SINGLE CHANNEL ECG

Solving identification problem is based on the comparison of the phase portraits of the current ECG of the person B_m being tested with pre-constructed reference ECGs of a limited group $B_1, \ldots, B_M$ of persons. Since, during physical or emotional stress, there are changes in certain parts of the ECG and in the wave T, in particular, we will analyze the most stable fragment of the phase portrait in what follows. Experiments have shown that this is a fragment corresponding to the ventricular complex QRS.

According to [25], to estimate the proximity of two phase portraits F_i and F_j, we calculate the Hausdorff distances

$$L_H(F_i, F_j) = \max\left\{ \max_{q_k \in F_i} \min_{q_r \in F_j} \rho(q_k, q_r),\ \max_{q_r \in F_j} \min_{q_k \in F_i} \rho(q_k, q_r) \right\}, \tag{1}$$

where $\rho(q_k, q_r) = \| q_k - q_r \|$ is the Euclidean distance between the normalized vectors $q_k = (z_k, \dot{z}_k) \in F_i^{(QRS)}$, $k = 1, \ldots, K_i^{(QRS)}$, and $q_r = (z_r, \dot{z}_r) \in F_j^{(QRS)}$, $r = 1, \ldots, K_j^{(QRS)}$, which belong to the QRS-fragments of the ith and jth phase portraits.

To construct a reference ECG PhP of the mth ($m = 1, \ldots, M$) representative of the group $B_1, \ldots, B_M$, register $W^{(m)}$ of his (her) ECGs at different times of the day and form the matrix $W^{(m)} \times W^{(m)}$ of Hausdorff distances (1) between the pairs of QRS-fragments of the corresponding phase portraits $F_1^{(m)}, \ldots, F_W^{(m)}$. The row number of this matrix, whose sum of elements is minimal, determines the reference phase portrait $F_0^{(m)}$, i.e.,

$$F_0^{(m)} = \arg \min_{1 \le i \le W^{(m)}} \sum_{j=1}^{W^{(m)}} L_H(F_i, F_j). \tag{2}$$

The decision in favor of the mth representative of the group is accepted only if

$$L_H(F_t, F_0^{(m)}) = \min_{1 \le \gamma \le M} L_H(F_t, F_0^{(\gamma)}) \wedge L_H(F_t, F_0^{(m)}) < \lambda^{(m)}, \tag{3}$$

where $L_H(F_t, F_0^{(\gamma)})$ are the Hausdorff distances between the current phase portrait F_t of the person being tested and all the reference phase portraits $F_0^{(1)}, \ldots, F_0^{(M)}$ defined according with (2), and $\lambda^{(m)}$ is a threshold, which is determined by the maximum element of the matrix of Hausdorff distances constructed at the stage of formation of $F_0^{(m)}$. The threshold $\lambda^{(m)}$ provides the necessary compromise between the probabilities of "missing a target" and "false alarm" in terms of identification of the mth representative of the group.

To estimate the reliability of the proposed approach, an experimental database of 3,133 ECGs of 167 different individuals was constructed. In 96.6% of cases (3,027 ECGs), the identification of persons according to rule (3) was correct.

Despite the high results, the proposed approach has limited possibilities. The fact is that the phase portraits of a single channel ECG of an individual have only minor differences (see Fig. 3). It is no exception that the ECGs of different people are generally almost identical, and the probability of such coincidences increases with increasing number in the group. It follows from the above that the construction of a reliable system of identification of a particular person by the ECG PhP in a rather large group can be considered only as a distant goal.

At the same time, as a short-term goal, we can formulate a simple but important task: to classify the ECG of an individual in certain classes (whose number can be set not very large) and verify the person by comparing his (her) current ECG PhP with the known class number of his (her) ECG PhP and the blood type belonging to this person. If the classes differ with a high probability, it is possible to claim a negative verification result and make appropriate decisions.

HUMAN VERIFICATION BY A PHASE PORTRAIT OF A SINGLE CHANNEL ECG

To develop a verification system, we define the characteristic types of ECG PhPs according to the available training sample, which contains the ECGs of different persons, for which the corresponding phase portraits

$$F = \{F_1, \ldots, F_Q\} \tag{4}$$

are constructed.

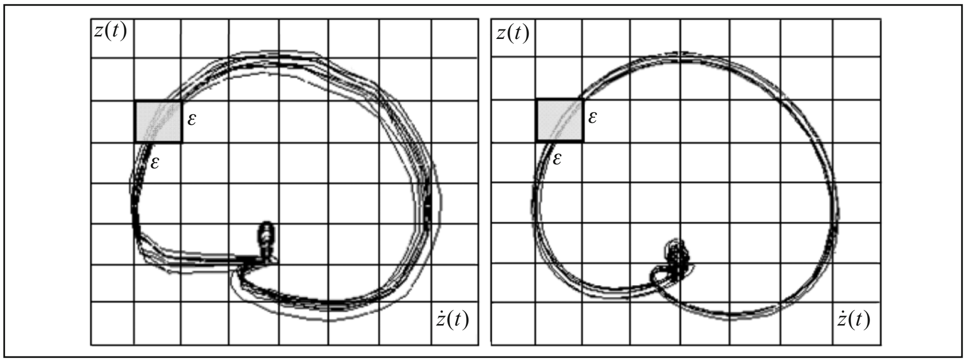

Fig. 5. Explanation of the method of estimating the distances between the ECG PhP images.

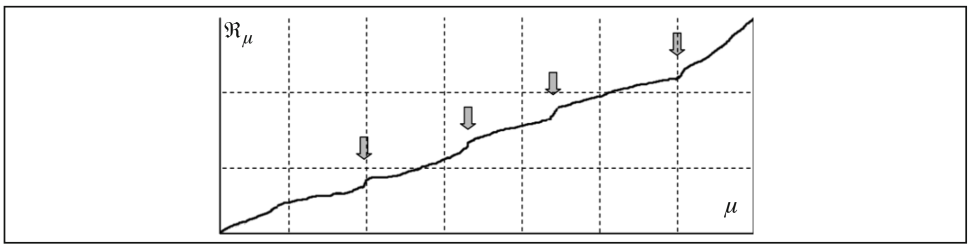

Fig. 6. Graph of ordered distances between ECG PhP clusters.

We will investigate the possibility of an alternative approach to estimating the proximity of the phase portraits $F_i \in F$ and $F_j \in F$, which is based on the analysis of their images. To do this, we calculate the absolute value of the difference between the functions $\Psi_i(x, y)$ and $\Psi_j(x, y)$, which characterize the number of black dots in the field of images of phase portraits $F_i \in F$ and belong to square cells with the sides ε in coordinates x and y of the phase plane (Fig. 5):

$$L_{ij} = \sum_{x,y} |\Psi_i(x, y) - \Psi_j(x, y)|. \tag{5}$$

Before calculating distances (5), we normalize the phase coordinates according to the formulas

$$z^*(t_k) = \frac{z(t_k) - \min\limits_{1\le k\le K} z(t_k)}{\max\limits_{1\le k\le K} z(t_k) - \min\limits_{1\le k\le K} z(t_k)}, \quad k = 1, \dots, K,$$

$$\dot{z}^*(t_k) = \frac{\dot{z}(t_k) - \min\limits_{1\le k\le K} \dot{z}^*(t_k)}{\max\limits_{1\le k\le K} \dot{z}^*(t_k) - \min\limits_{1\le k\le K} \dot{z}^*(t_k)}, \quad k = 1, \dots, K,$$

that provide the compliance with the conditions $z^*(t_k) \in [0, 1]$ and $\dot{z}^*(t_k) \in [0, 1]$.

Let us form the square matrix

$$\Lambda = \begin{pmatrix} L_{11} & L_{12} & \dots & L_{1Q} \\ L_{21} & L_{22} & \dots & L_{2Q} \\ & \dots & & \\ L_{Q1} & L_{Q2} & \dots & L_{QQ} \end{pmatrix} \tag{6}$$

of distances (5) between all pairs of phase portrait images of the training sample, which was used to cluster the ECG PhPs.

The element of matrix (6), which corresponds to the maximum distances L_{ij}, determines the first pair of ECG PhP clusters. To determine the rest of the clusters, we arrange the elements of matrix (6) row corresponding to the first cluster in ascending order. The presence of “jumps” in the sequence $\Re_\mu$, $\mu = 1, \dots, Q$, of ordered distances (Fig. 6) determines the boundaries between the ECG PhP clusters. Such jumps can be easily found using a simple computational procedure.

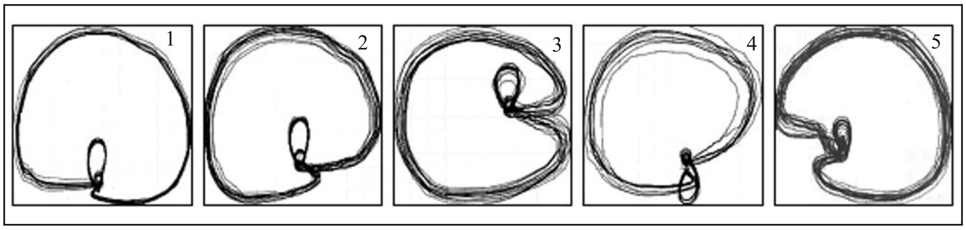

Fig. 7. Characteristic ECG PhP clusters.

It is experimentally established that according to the criterion of the maximum ratio of interclass and intraclass distances, the optimal cell size should be $\varepsilon = 0.125H$, where H is the size of the field of the ECG PhP images. In this case, the image field is covered by 64 square cells.

According to the analysis of 300 ECGs of 115 different people, $J = 5$ characteristic clusters of ECG PhPs (Fig. 7) were identified.

Let us choose a subset of phase portraits corresponding to the νth cluster ($\nu = 1, \dots, 5$) from set (4) and define the reference cluster as follows:

$$\Im_0^{(\nu)} = \arg\min_{1 \le i \le Q_\nu} \sum_{j=1}^{Q_\nu} L_{ij}^{(\nu)}, \tag{7}$$

where $L_{ij}^{(\nu)}$ is distance (6) between the pairs of ECGs of the νth cluster in the training sample and Q_ν is the number of ECG PhPs of the νth cluster.

Person verification is performed according to the reference clusters $\Im_0^{(1)}, \dots, \Im_0^{(5)}$ constructed according to (7). Such information (login of the person being tested) can be obtained by scanning the microprocessor chip of a biometric passport, credit card, or other personal document.

The comparison is made by calculating the distances

$$L(F_t, \Im_0^{(\nu)}) = \sum_{x,y} |\Psi^{(t)}(x, y) - \Psi^{(\nu)}(x, y)|, \ \nu = 1, \dots, 5, \tag{8}$$

between the images $\Psi^{(t)}(x, y)$ of the phase portrait of the current ECG of the person being tested, and five reference images $\Im_0^{(1)}, \dots, \Im_0^{(5)}$ that are stored in the database of the access system.

The verification system determines the reference number $\Im_0^{(t)}$, which has minimum distance (8) from the image of the phase portrait F_t of the current ECG automatically, i.e.,

$$\Im_0^{(t)} = \arg\min_{1 \le \nu \le 5} L(F_t, \Im_0^{(\nu)}). \tag{9}$$

We conclude that the verification is considered to be successful only when the reference cluster $\Im_0^{(t)}$ determined in accordance with (9) coincides with the entered login $\Im_0^{(p)}$, i.e., $\Im_0^{(t)} = \Im_0^{(p)}$.

Otherwise, the verification is not considered to be successful.

When testing the proposed approach on the examination sample containing 204 ECG recordings of 62 different individuals, there was only one failed verification (0.5%).

PROSPECTIVE RESEARCH

In addition to the considered methods of estimating the proximity of ECG PhP, which are based on formulas (1) and (5), we can construct other procedures that have the properties of metrics. Thus, a natural question arises: which of the metrics is more efficient?

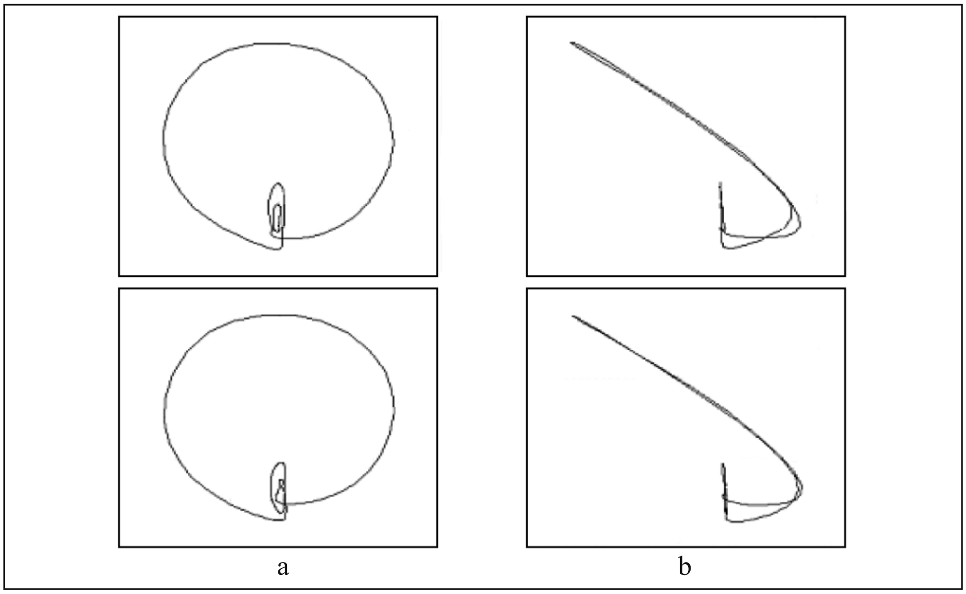

Fig. 8. Projections on the plane of three-dimensional phase portraits of two people for different projection angles.

Consider the formal scheme of reasonable choice of the degree of proximity of the ECG PhPs by training sample, which contains G phase portraits $F_1, \dots, F_G$. Assume that each ith phase portrait belongs to one of the different $M \geq 2$ persons from the group $B = \{B_1, \dots, B_M\}$, and for each person there are several phase portraits, i.e., $G >> M$.

Let us have $N \geq 2$ various measures of proximity $S_1, \dots, S_N$. Each nth measure S_n $(1 \leq n \leq N)$ generates its own decision rule R_n according to which the analyzed phase portrait $F_t \in \{F_1, \dots, F_G)$ belongs to one of the representatives of the group $B = \{B_1, \dots, B_M\}$.

Consider the function

$$\Omega_t^{(n)} = \begin{cases} 0 & \text{if the decision is true,} \\ 1 & \text{if the decision is false,} \end{cases} \tag{10}$$

which determines the losses caused by the wrong decision made according to the phase portrait F_t taking into account the measure S_n, $1 \leq n \leq N$. Using function (10) allows estimating the quality of each n-decision rule as follows:

$$\Omega^{(n)} = \sum_{t=1}^{G} \Omega_t^{(n)}$$

and consider the measure S_{n0}, $1 \leq n_0 \leq N$, which satisfies the condition

$$n_0 = \arg \min_{1 \leq n \leq N} S^{(n)} \tag{11}$$

the best.

Relation (11) makes it possible to choose the best measure of proximity of the ECG PhP.

It is also advisable to investigate the possibility of increasing the reliability of decisions using the transition to the ECG PhP in three-dimensional space with the coordinates $z(t)$, $\dot{z}(t)$, and $\ddot{z}(t)$. The first experiments showed that even if the two-dimensional ECG PhPs of different people almost coincide (Fig. 8a), then during the transition into three-dimensional space, it is possible to detect some differences of supposedly identical phase portraits for certain design angles (Fig. 8b).

The analysis of these problems on a representative sample of observations allows extending the capabilities of biometric systems for solving practical problems in various fields.

CONCLUSIONS

An approach to construction of biometric systems based on the analysis of a phase portrait of the single channel electrocardiogram of a person being tested is improved. Long-term observations have shown that the ECG PhP, as well as fingerprints, has individual features (see Fig. 3) unchanged over a fairly long period of time (see Fig. 4).

Decision rules have been constructed to provide the identification and verification (authentication) of a person. The test results confirmed the efficiency of the proposed approaches and relatively high reliability of decisions: 96.6% of correct decisions in the process of human identification on 3,133 ECG recordings in a group of 167 people and 99.5% of correct decisions in the process of human verification on 204 ECG recordings of 62 different people.

Prospects for further research aimed at improving the reliability of decisions are determined.

Biometric systems based on the analysis of the phase portrait of a single channel ECG can be used for both stand-alone system and as a part of available systems. For example, if information about the ECG PhP class of the owner is entered in the microprocessor chip of the biometric passport, then when crossing the border, he (she) can be verified not only by fingerprint, as they do now, but also using additional information that will increase the reliability of decisions.

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